Gujarat Secondary and Higher Secondary Education Board, Gandhinagar Standard – 12 Diagnostic Test

Subject : Maths (050)		Total Marks : 80	
M	edium : English	Time : 3 Hours	
<u>Section – A</u>			
1)	Answer the following questions (Each carries	1 mark) [24]	
i.	If $n(p(p(A))) = 4$, then A = (a) { ϕ } (b) {1,2} (c) ϕ (d) U		
ii.	Write the set $\{x \in \mathbb{R} / -4 < x \le 6\}$ in interval form.		
iii.	If $\left(\frac{x}{3}+1, y-\frac{2}{3}\right)=\left(\frac{5}{3}, \frac{1}{3}\right)$, then find the values of x and y.		
iv.	The range of function f : R \rightarrow R, f(x) = x ² + 2 is		
ν.	Find the radian measure corresponding to the degree measure 25°.		
vi.	Find the value of Sin 75 ⁰ .		
vii.	The complex number $3(7 + i7) + i(7 + i7) = a + ib$	ber 3(7 + i7) + i(7 + i7) = a + ib then a =	
viii.	The roots of the equation $x^2 - x + 2 = 0$ are in (a) Complex set C (b) Real set R (c) Irrational set R – Q (d) Integer set Z		
ix.	If ${}^{n}C_{2} = {}^{n}C_{8}$, then $n = $		
х.	What is the number of ways of choosing 4 cards of the same suits from a pack of 52 cards?		
xi.	Find n th term of the sequence 7, 9, 11, 13		
xii.	If A and G are A.P. and G.P. respectively of the numbers 4 and 16, then $A - G = $		
xiii.	Write the equation of the line passing through the point (-4, 3) with a slope $\frac{1}{2}$		
xiv.	Reduce the eqation $\sqrt{3}x + y - 8 = 0$ into normal form.		
xv. xvi.	Find the centre and radius of the circle $x^2 + y^2 - 4x - 8y - 45 = 0$. Find the equation of the parabola with vertex at (0,0) and focus at (0,2).		
vii.	Find $\frac{d}{dx}(ax^2 + b)^2$, where a and b are constants.		
	ux .		
viii.	If $f(x) = \frac{x+1}{x}$, $x \neq 0$, then find f'(1).		
xix.	$\lim_{x \to \frac{\pi}{2}} \left(\frac{\tan 2x}{x - \frac{\pi}{2}} \right) = \underline{\qquad}$		
xx. xxi.	 (a) 2 (b) -1 (c) 1 (d) -2 If p(A) = 0.99, then p(A') = What is the probability of the event "Having 5 Sun 	days in the month of	
	January 2021?	-	

- **xxii.** If $\binom{n}{1} + \binom{n}{2} + \binom{n}{3} + \cdots \binom{n}{n} = 31$, then n =_____.
- **xxiii.** Find the value of $sin^2 \frac{\pi}{6} + cos \frac{\pi}{3} tan \frac{\pi}{4}$.

xxiv. Evaluate : $\sum_{r=1}^{11} (2 + 3^r)$

Section – B

- Answer any 11 questions from given questions (Each carries 2 marks)
 - Let A : { x : x is a natural number }
 B : { x : x is an even natural number } and
 C : { x : x is a prime number }
 Find (i) A B (ii) C A
 - **3.** If A = {1, 2, 3, 4, 6} and relation R = {(a,b) : a, $b \in A$, b is divisible by a}, then (1) Write R in roster form. (2) Find domain and range of R.
 - **4.** In a circle of diameter 40 cm, the length of a chord is 20 cm. Find the length of minor arc of the chord.
 - 5. $\tan x = \frac{-5}{12}$, x lies in second quardrant. Find the values of other five trigonometric functions..

6. Express
$$z = \frac{5+\sqrt{2}i}{1-\sqrt{2}i}$$
 in a + ib form.

- 7. Solve the equation : $\sqrt{2}x^2 + x + \sqrt{2} = 0$
- 8. If ${}^{5}P_{r} = 2 \cdot {}^{6}P_{r-1}$, then find the value of r.
- **9.** In how many ways can a team of 3 boys and 3 girls be selected from 5 boys and 4 girls?
- 10. Insert two numbers between 3 and 81 so that the resulting sequence is G.P.
- **11.** If three points (h,0), (a,b) and (0,k) lies on a same line, show that $\frac{a}{b} + \frac{b}{b} = 1$.
- **12.** What are the points on the y-axis whose distance from the line $\frac{x}{3} + \frac{y}{4} = 1$ is 4 units?
- **13.** Find the equation of the circle passing through (0,0) and making intercepts a and b on the coordinate axis.
- **14.** Find the equation of hyperbola whose vertices $(\pm 2, 0)$ and foci $(\pm 3, 0)$.
- **15.** Find $\lim_{x \to 0} \frac{x(e^{x}-1)}{1 \cos x}$.
- **16.** A coin is tossed twice. What is the probability that atleast one tail occurs?
- **17.** If P(A) = 0.54, P(B) = 0.69, $P(A \cap B) = 0.35$, then find $P(A' \cap B')$ and $P(B \cap A')$.

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Section – C

Answer any 8 questions from the following questions. (Each carries 3 marks)

- 18. If U = {1,2,3,4,5,6,7,8,9}, A = {2,4,6,8} and B={2,3,5,7}, then verify that... (1) $(A \cup B)' = A' \cap B'$ (2) $(A \cap B)' = A' \cup B'$
- 19. Let $f = \{(1,1), (2,3), (0,-1), (-1,-3)\}$ be a linear function from z to z. Find f(x).
- 20. Prove that : $\cos 6x = 32\cos^6 x 48\cos^4 x + 18\cos^2 x 1$
- 21. Find the modulus and the argument of $z = -\sqrt{3} + i$.
- 22. How many 4 digit numbers can be formed using the digits 1, 2, 3, 4, 5, if no digit is repeated? How many of these will be even?
- 23. In how many ways can the letters of the word PERMUTATIONS be arranged, if the...
 - (i) Words start with P and end with S.
 - (ii) All the vowels are together.

24. If $\frac{a^{n}+b^{n}}{a^{n-1}+b^{n-1}}$ is the arithmetic mean of a and b, then find the value of n.

25. If the measure of angle between two lines is θ and tan $\theta = \frac{1}{3}$ and the slope of a line is double of the slope of another line, then find the slope of two lines.

26. Find the coordinates of the foci, the eccentricity and the length of latus $u^2 = u^2$

rectum of the ellipse
$$\frac{x^2}{25} + \frac{y^2}{100} = 1$$

27. If
$$y = \frac{Sin(x+a)}{Cos x}$$
, then find $\frac{dy}{dx}$.

28. If
$$y = (x + \sec x)(x - \tan x)$$
, then find $\frac{dy}{dx}$.

- 29. Three coins are tossed once. Find the probability of getting...
 - (1) atleast two heads.
 - (2) atmost two tails.

30.

Section – D

Answer any two from the following questions. (Each carries 5 marks)

Prove that,
$$Cos^{2}x + Cos^{2}\left(x - \frac{\pi}{3}\right) + Cos^{2}\left(x + \frac{\pi}{3}\right) = \frac{3}{2}$$

31. Find the sum of n terms of the sequence 8, 88, 888, 8888,

32. If
$$f(x) = \begin{cases} mx^2 + n \ ; \ x < 0 \\ nx + m \ ; \ 0 \le x \le 1 \\ nx^3 + m \ ; \ x > 1 \end{cases}$$

for what integers m and n does both $\lim_{x\to 0} f(x)$ and $\lim_{x\to 1} f(x)$ exists?

[10]

[24]